# THOUGHTS ON APPLIED MATHEMATICS

Applied Mathematics consists of two words: 'applied' and 'mathematics'. In general, Applied Mathematics also include two areas. One area is the Mathematics related to Application, which may be called APPLICABLE MATHEMATICS. This a subset of traditional mathematics. The other area is the APPLICATION OF MATHEMATICS, i.e., the study and solution of scientific and engineering problems using mainly mathematics as the tool. This is not a subset of the traditional mathematics and is beyond traditional mathematics.

## Process of Applied Mathematics

These two areas of applied mathematics are related through the Process of Applied Mathematics, which involves 5 steps:

1.Observations; Experimentations; Collection and Organization of Information.

2.Establishment of Mathematical Model.

3.Create New Mathematics and Mathematical Tools; Extension and Development of Known Mathematics.

4.Solving the Mathematical Problem.

5.Check with the Experiments and Observations.

The best model applied mathematician is Newton. In order to understand the motion of heavenly bodies, he took the following 5 steps:

1.Organnization of existing observation data and personal astronomical observations.

2.Laws of mechanics and law of gravitation.

3.Invention of calculus.

4.Solving the problem of motion of heavenly bodies.

5.Agreement with observation.

Newton was a rare genius. Scientific development in Newton's time was relatively simple in scope. Hence Newton could single-handedly complete an entire Process of Applied Mathematics. Most of us cannot compare with Newton. The diversity and complexity of modern scientific development force us to be specialists in a narrow discipline. Even for problems of rather small scope, few persons can complete a Process of Applied Mathematics all by himself. Thus we shall consider those scientists who undertake some part of the work in the Process of Applied Mathematics, and who are keenly aware of the underlying Process, "applied mathematicians".

The APPLICABLE MATHEMATICS are concerned with the steps 3 and 4 in the Process. Nowadays, most so-called applied mathematicians in the US, and almost all those in China are doing applicable mathematics. Unfortunately, most of them do not have the Process of Applied Mathematics in mind. Thus they miss the essence of the problem and dwell too much on unimportant mathematical technicalities. That is why Hardy would term it trivial mathematics and earned it the scorn from pure mathematicians.

## Any Need of Applied Mathematics Program?

The fact that, whether independent or not, almost every university has applied mathematics program shows that there must be needs for it. Scientists and engineers know that mathematics is useful. They wish mathematicians will help them. Departments of mathematics also need to demonstrate to the public, e.g., the legislature for public universities, that they are doing things relevant to welfare of the society. So there is no question about the need of applied mathematics programs.

But it is largely because of the societal pressure that mathematics departments set up applied mathematics program. Someone has to teach 'useful' courses to scientists and engineers, but the hearts of those so engaged are not in them. Someone need to be involved in research in 'applied mathematics'. They can only do the "applicable mathematics' because they are mathematicians and do not know the science. The mathematical contents or level are not that high usually, according to the main-stream mathematical standard, and they would be looked down as second class citizens by the pure mathematicians.

This unsatisfactory situation can be resolved if there is an applied mathematics program with genuine applied mathematicians. The genuine applied mathematicians will appreciate the relevance of the mathematics they teach and the intricacies of the new developments. They are qualified scientists or engineers, and what they study are beyond traditional mathematics. They won't be second class citizens.

It is very common to see in many universities, on the one hand the so-called applied mathematicians in the mathematics department are unhappy because of their second class citizenship, on the other hand, the engineers and science students are complaining that they can learn nothing useful from the mathematicians. Recently, because of the rapid development of computers and computation softwares, engineers and scientists are not complaining as much. They have simply stopped learning mathematics and just stuff their problems into ready made software black boxes, and grind out beautiful graphs and movies. I do not think it will work in the long run. Sooner or later we shall be in need of more good genuine applied mathematicians.

## Where to Place Applied Mathematics?

At Brown University, University of Cambridge and Caltech, Applied Mathematics programs are independent departments, while at MIT and NYU, they are just a component in the mathematics department. They are all good applied mathematics programs. Whether this way or that way has nothing to do with the size, because the Caltech's program is quite small with only 6 to 8 faculty members. It also has nothing to do with the contents, because MIT's applied mathematics program emphasizes very much on scientific subject matters.

Presumably, the advantage of placing the applied mathematics program in the mathematics department is to provide close interaction between practitioners of pure thoughts and worldly theoreticians. They should benefit each other. Natural phenomena are always a source of inspiration for new mathematical ideas. On the other hand, deeper mathematical understandings also often open up new avenues for scientific explorations. However because of the difference in personal background and nature of study, there is also great difference in value judgments. It is not easy to resolve these contradictions.

A mutual respect and appreciation from both sides is necessary for the desirable coexistence of pure and Applied mathematicians in the same department. Perhaps an informal autonomy for each group should be in place for faculty development and curricular planning.

How to Train Applied Mathematicians?

At HKUST, I think we are on the right track with various options in the Mathematics Department to train applied mathematicians at the undergraduate level. It would be even more desirable if other departments have the same vision to encourage students to take much stronger dose of mathematics courses. Because their students in general are more likely to have their hearts in the discipline they have chosen, and hence have greater potential to establish right mathematical model.

At higher level of learning, applied mathematicians are usually grown into their roles. Research applied mathematicians are just those mathematically inclined, broad-minded theoretical scientists. Often they are not the product of formal applied mathematics department.

The focus of various applied mathematics departments are defined essentially by the activities of a core group. Thus at Brown in early days, the study of plasticity initiated by Prager developed into a center of solid mechanics. At Cambridge as well as Caltech, fluid mechanics became the foci under the leadership of Taylor and Von Karman respectively.

Towards New Disciplines.

Applied Mathematics programs have been parent programs for quite a few new disciplines. Computer Science departments in many universities have been the offspring of the applied mathematics programs. Statistics is another example.

Econometrics and plasma physics have also been actively pursued in the early days by applied mathematicians. When a scientific or technological discipline is about ready to bear fruits by intensive mathematical studies, applied mathematicians are often the pioneers for its development, because they have the right temperament and environment.

Nowadays, the discipline of scientific computation, which is a central activity of every applied mathematics program, probably will become an independent department if not already. For the future, life science is bound to be intensively explored by applied mathematicians. It is still largely a virgin soil waiting for breakthroughs. It is still in the Kepler era waiting for Newton to come. Genome, Protein Structure, and Neural Science are some areas that come to my mind.

Some Recent Prominent Applied Mathematicians.

Von Karman: In 1950s and 60s, most top fluid dynamics group in the US were led by his students. Apparently he just identified the important problems in aeronautics and advised students to go for it. He did not put his name on the published works of the students like what people nowadays usually do. The central problem can be simply stated: to make an object heavier than the air fly.

G.I.Taylor: A remarkable genius who often picks a simple-looking problem, and solves it with relatively simple means, but opens up a vast area of study.

Prager: Solid mechanics used to deal with elasticity. Materials are considered failure beyond the yielding point. That is when plasticity theory takes over. Prager was the pioneer in the study of plasticity. He also founded the first applied mathematics department in the US at Brown University.

Samuelson: An economist with strong mathematical background, he opened up the discipline of econometrics.

Spitzer: An astrophysicist who saw the importance of the study of the ionized gases which combine the study of fluid mechanics and electrodynamics. He was the pioneer in the study of plasma physics.

Chandrasekhar: A remarkable and prolific astrophysicist and fluid dynamist in the classical tradition.

I sometimes just wonder where we can find such applied mathematicians now.

Zhou Pei-Yuan Center for Applied Mathematics.

What we would like to do:

1. We emphasize the Application of Mathematics, keenly aware of the Process of Applied Mathematics.

2. While we would like to influence the mathematics department with our view of applied mathematics, we do not want to be entangled in the internal politics of the mathematics department. Therefore we shall stay independent. Hopefully, if we are successful, they may follow our example. If Tsinghua will reform, then the nation will follow.

3. We would like go into the area of life science.

4. But there are practically no such mathematical biologist available to us at Tsinghua. So we have to train ourselves, and it takes time.

5. Basically, we just offer an environment, a reasonably good working environment for people with patience and dedication to explore together.